The angular velocity of the apsidal rotation in binary stars

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Abstract

The shape of a rotating star consisting of equilibrium plasma is considered. The velocity of apsidal rotation of close binary stars (periastron rotation) which depends on the star shapes is calculated. The obtained estimations are in a good agreement with the observation data of the apsidal motion in binary systems.

1 Introduction

The apsidal motion (periastron rotation) of close binary stars is result of a their non-Keplerian moving which originates from the non-spherical form of stars. This non-sphericity has been produced by a rotation of stars about their axes or by their mutual tidal effect. The second effect is smaller usually and it can be neglected. Following to the traditional approach to explanation of this effect, one needs to suppose the existence of a concentration of mass inside central part of stars. To reach an agreement between the measuring data and calculations, it is usually necessary to assume that the density of substance at the central region of a star is a hundred times more than a mean density of the star [1].

As it was shown earlier [3], almost the full mass of a star is concentrated in its plasma core at a permanent density. Therefor the effect of periastron rotation of close binary stars must be reviewed with the account of a change of forms of these star cores.

According to [1]-[2] the ratio of the angular velocity ω of rotation of periastron which is produced by the rotation of a star about its axis with the angular velocity Ω is

$$\frac{\omega}{\Omega} = \frac{3}{2}\frac{(I_A - I_C)}{Ma^2} \qquad (1)$$

where I_A and I_C are the moments of inertia relatively to principal axes of the ellipsoid. Their difference is

$$I_A - I_C = \frac{M}{5}(a^2 - c^2), \tag{2}$$

where a and c are the equatorial and polar radii of the star.

Thus we have

$$\frac{\omega}{\Omega} \approx \frac{3}{10}\frac{(a^2 - c^2)}{a^2}. \tag{3}$$

2 The equilibrium form of the core of a rotating star

In the absence of rotation the equilibrium equation of plasma inside star is [3]

$$\gamma \mathbf{g}_G + \rho_G \mathbf{E}_G = 0 \tag{4}$$

where γ,$\mathbf{g}_G$, ρ_G and $\mathbf{E}_G$ are the substance density the acceleration of gravitation, gravity-induced density of charge and intensity of gravity-induced electric field ($div\ \mathbf{g}_G = 4\pi\ G\ \gamma$, $div\ \mathbf{E}_G = 4\pi\rho_G$ and $\rho_G = \sqrt{G}\gamma$).

One can suppose, that at a rotation, under action of a rotational acceleration $\mathbf{g}_\Omega$, an additional electric charge with density ρ_Ω and electric field $\mathbf{E}_\Omega$ can exist, and the equilibrium equation obtains the form:

$$(\gamma_G + \gamma_\Omega)(\mathbf{g}_G + \mathbf{g}_\Omega) = (\rho_G + \rho_\Omega)(\mathbf{E}_G + \mathbf{E}_\Omega), \tag{5}$$

where

$$div\ (\mathbf{E}_G + \mathbf{E}_\Omega) = 4\pi(\rho_G + \rho_\Omega) \tag{6}$$

or

$$div\ \mathbf{E}_\Omega = 4\pi\rho_\Omega. \tag{7}$$

We can look for a solution for electric potential in the form

$$\varphi = C_\Omega\ r^2(3cos^2\theta - 1) \tag{8}$$

or in Cartesian coordinates

$$\varphi = C_\Omega(3z^2 - x^2 - y^2 - z^2) \tag{9}$$

where C_Ω is a constant.
Thus

$$E_x = 2\ C_\Omega\ x,\ \ E_y = 2\ C_\Omega\ y,\ \ E_z = -4\ C_\Omega\ z \tag{10}$$

and

$$div\ \mathbf{E}_\Omega = 0 \tag{11}$$

and we obtain the important equations:

$$\rho_\Omega = 0; \tag{12}$$

$$\gamma g_\Omega = \rho \mathbf{E}_\Omega. \tag{13}$$

Since a centrifugal force must be contra-balanced by the electric force

$$\gamma\ 2\Omega^2\ x = \rho\ 2C_\Omega\ x \tag{14}$$

and

$$C_\Omega = \frac{\gamma\ \Omega^2}{\rho} = \frac{\Omega^2}{\sqrt{G}} \tag{15}$$

The potential of a positively uniformly charged ball is

$$\varphi(r) = \frac{Q}{R}\Big(\frac{3}{2} - \frac{r^2}{2R^2}\Big) \tag{16}$$

The negative charge on the surface of a sphere induces inside the sphere the potential

$$\varphi(R) = -\frac{Q}{R} \tag{17}$$

where accordingly to Eq.(4) $Q = \sqrt{G}M$, and M is the mass of the star.
Thus the total potential inside the considered star is

$$\varphi_\Sigma = \frac{\sqrt{G}M}{2R}\left(1 - \frac{r^2}{R^2}\right) + \frac{\Omega^2}{\sqrt{G}} r^2 (3cos^2\theta - 1) \tag{18}$$

Since the electric potential must be equal to zero on the surface of the star, at $r = a$ and $r = c$

$$\varphi_\Sigma = 0 \tag{19}$$

and we obtain the equation which describes the equilibrium form of the core of a rotating star (at $\frac{a^2-c^2}{a^2} \ll 1$)

$$\frac{a^2 - c^2}{a^2} \approx \frac{9}{2\pi}\frac{\Omega^2}{G\gamma}. \tag{20}$$

3 The angular velocity of the apsidal rotation

Taking into account of Eq.(20) we have

$$\frac{\omega}{\Omega} \approx \frac{27}{20\pi}\frac{\Omega^2}{G\gamma} \tag{21}$$

If both stars of a close pair induce a rotation of periastron, this equation transforms to

$$\frac{\omega}{\Omega} \approx \frac{27}{20\pi}\frac{\Omega^2}{G}\left(\frac{1}{\gamma_1} + \frac{1}{\gamma_2}\right), \tag{22}$$

where γ_1 and γ_2 are densities of star cores.

The equilibrium density of star cores is known [3]:

$$\gamma = \frac{16}{9\pi^2}\frac{A}{Z}m_p\frac{(Z+1)^3}{a_B^3}, \tag{23}$$

where A and Z are the mass number and charge of nuclei of plasma, m_p is proton mass, and the Borh radius is

$$a_B = \frac{\hbar^2}{m_e e^2}. \tag{24}$$

If we introduce the period of ellipsoidal rotation $P = \frac{2\pi}{\Omega}$ and the period of the rotation of periastron $U = \frac{2\pi}{\omega}$, we obtain from Eq.(21)

$$\frac{P}{U}\left(\frac{P}{T}\right)^2 \approx \sum_1^2 \xi_i, \tag{25}$$

where

$$T = \sqrt{\frac{243\ \pi^3}{80}}\ \tau_0 \approx 10\tau_0, \tag{26}$$

$$\tau_0 = \sqrt{\frac{a_B^3}{G\ m_p}} \approx 7.7 \cdot 10^2 sec \tag{27}$$

and

$$\xi_i = \frac{Z_i}{A_i(Z_i+1)^3}. \tag{28}$$

4 The comparison of the calculated angular velocity of the periastron rotation with observations

Because the substance density (Eq.(23)) is depending approximately on the second power of the nuclear charge, the periastron moving of stars consisting of heavy elements will fall out from the observation as it is very slow. Practically the obtained equation (25) shows that it is possible to observe the periastron rotation of a star consisting of light elements only.

The value $\xi = Z/[A(Z+1)^3]$ is equal to 1/8 for hydrogen, 0.0625 for deuterium, $1.85 \cdot 10^{-2}$ for helium. The resulting value of the periastron rotation of double stars will be the sum of separate stars rotation. The possible combinations of a couple and their value of $\sum_1^2 \xi_i$ for stars consisting of light elements is shown in Table 1.

star1 composed of	star2 composed of	$\xi_1 + \xi_2$
H	H	.25
H	D	0.1875
H	He	0.143
H	hn	0.125
D	D	0.125
D	He	0.0815
D	hn	0.0625
He	He	0.037
He	hn	0.0185

Table 1.

There "hn" notation in Table 1 indicates that the second component of the couple consists of heavy elements or it is a dwarf.

The periods U and P are measured for few tens of close binary stars. The data of these measurement is summarized in the Table 2. In this table U is the period of the periastron rotation in years, P is the period of the orbital rotation in astronomical days. $M_1/M_\odot$ and $M_2/M_\odot$ are masses of the first and the second star over the solar mass, $R_1/R_\odot$ and $R_2/R_\odot$ are the first star radius and the second star radius over the solar radius, T_1 and T_2 are the surface temperatures of the first and the second star, $a/R_\odot$ is the orbital radius of the couple over solar radius. All these data and references was given to us by Dr.Khaliullin K.F. (Sternberg Astronomical Institute) [4].

One can compare our calculation with the data of these measurements. The distribution of close binary stars on value of $(P/U)(P/T)^2$ is shown on Fig.1 in logarithmic scale. The lines mark the values of parameters $\sum_1^2 \xi_i$ for different pairs of binary stars. It can be seen that the calculated values the periastron rotation for stars composed by light elements which is summarized in Table 1 are in the good agreement with separate peaks of measured data. It confirms that our approach to interpretation of this effect and is adequate to produce the satisfactory accuracy of estimations.

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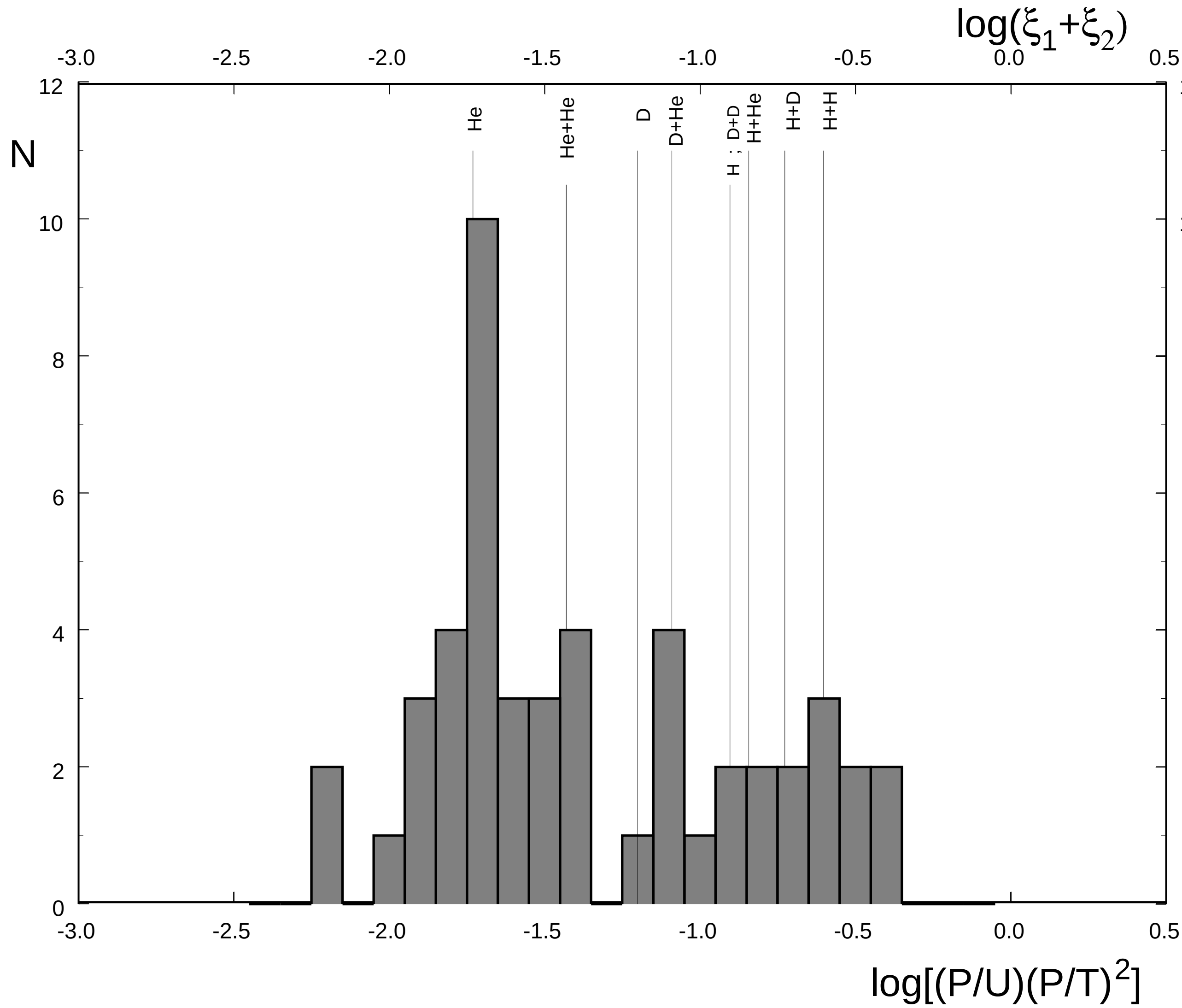

Figure 1: The distribution of binary stars on value of $(P/U)(P/T)^2$.

N	Name of star	U	P	$M_1/M_\odot$	$M_2/M_\odot$	$a/R_\odot$	$R_1/R_\odot$	$R_2/R_\odot$	T_1	T_2
1	BW Aqr	5140	6.720	1.48	1.38	21.26	1.803	2.075	6100	6000
2	V 889 Aql	23200	11.121	2.40	2.20	34.85	2.028	1.826	9900	9400
3	V 539 Ara	150	3.169	6.24	5.31	20.51	4.512	3.425	17800	17000
4	AS Cam	2250	3.431	3.31	2.51	17.21	2.580	1.912	11500	10000
5	EM Car	42	3.414	22.80	21.40	33.74	9.350	8.348	33100	32400
6	GL Car	25	2.422	13.50	13.00	13.28	4.998	4.726	28800	28800
7	QX Car	361	4.478	9.27	8.48	29.81	4.292	4.054	23400	22400
8	AR Cas	922	6.066	6.70	1.90	28.66	4.591	1.808	18200	8700
9	IT Cas	404	3.897	1.40	1.40	14.68	1.616	1.644	6450	6400
10	OX Cas	40	2.489	7.20	6.30	18.30	4.690	4.543	23800	23000
11	PV Cas	91	1.750	2.79	2.79	10.83	2.264	2.264	11200	11200
12	KT Cen	260	4.130	5.30	5.00	23.56	4.028	3.745	16200	15800
13	V 346 Cen	321	6.322	11.80	8.40	39.16	8.263	4.190	23700	22400
14	CW Cep	45	2.729	11.60	11.10	23.32	5.392	4.954	26300	25700
15	EK Cep	4300	4.428	2.02	1.12	16.61	1.574	1.332	10000	6400
16	α Cr B	46000	17.360	2.58	0.92	42.81	3.314	0.955	9100	5400
17	Y Cyg	48	2.997	17.50	17.30	28.54	6.022	5.680	33100	32400
18	Y 380 Cyg	1550	12.426	14.30	8.00	63.51	17.080	4.300	20700	21600
19	V 453 Cyg	71	3.890	14.50	11.30	30.74	8.607	5.410	26600	26000
20	V 477 Cyg	351	2.347	1.79	1.35	10.88	1.567	1.269	8550	6500
21	V 478 Cyg	26	2.881	16.30	16.60	27.29	7.422	7.422	29800	29800
22	V 541 Cyg	40000	15.338	2.69	2.60	45.24	2.013	1.900	10900	10800
23	V 1143 Cyg	10300	7.641	1.39	1.35	22.83	1.440	1.226	6500	6400
24	V 1765 Cyg	1932	13.374	23.50	11.70	77.64	19.960	6.522	25700	25100
25	DI Her	29000	10.550	5.15	4.52	43.10	2.478	2.689	17000	15100
26	HS Her	92	1.637	4.25	1.49	10.46	2.709	1.485	15300	7700
27	CO Lac	44	1.542	3.13	2.75	10.13	2.533	2.128	11400	10900
28	GG Lup	101	1.850	4.12	2.51	13.22	2.644	1.917	14400	10500
29	RU Mon	348	3.585	3.60	3.33	18.78	2.554	2.291	12900	12600
30	GN Nor	500	5.703	2.50	2.50	22.96	4.591	4.591	7800	7800
31	U Oph	21	1.677	5.02	4.52	12.59	3.311	3.110	16400	15200
32	V 451 Oph	170	2.197	2.77	2.35	12.25	2.538	1.862	10900	9800
33	β Ori	228	5.732	19.80	7.50	40.56	14.160	8.072	26600	17800
34	FT Ori	481	3.150	2.50	2.30	15.24	1.890	1.799	10600	9500
35	AG Per	76	2.029	5.36	4.90	14.65	2.995	2.606	17000	17000
36	IQ Per	119	1.744	3.51	1.73	10.58	2.445	1.503	13300	8100
37	ζ Phe	44	1.670	3.93	2.55	11.04	2.851	1.852	14100	10500
38	KX Pup	170	2.147	2.50	1.80	11.38	2.333	1.593	10200	8100
39	NO Pup	37	1.257	2.88	1.50	8.01	2.028	1.419	11400	7000
40	VV Pyx	3200	4.596	2.10	2.10	18.76	2.167	2.167	8700	8700
41	YY Sgr	297	2.628	2.36	2.29	13.37	2.196	1.992	9300	9300
42	V 523 Sgr	203	2.324	2.10	1.90	11.71	2.682	1.839	8300	8300
43	V 526 Sgr	156	1.919	2.11	1.66	10.11	1.900	1.597	7600	7600
44	V 1647 Sgr	592	3.283	2.19	1.97	14.94	1.832	1.669	8900	8900
45	V 2283 Sgr	570	3.471	3.00	2.22	16.72	1.957	1.656	9800	9800
46	V 760 Sco	40	1.731	4.98	4.62	12.89	3.015	2.642	15800	15800
47	AO Vel	50	1.585	3.20	2.90	11.41	2.623	2.954	10700	10700
48	EO Vel	1600	5.330	3.21	2.77	23.29	3.145	3.284	10100	10100
49	α Vir	140	4.015	10.80	6.80	27.64	8.097	4.394	19000	19000
50	DR Vul	36	2.251	13.20	12.10	21.21	4.814	4.369	28000	28000

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